Segmented telescopes co-phasing using Pyramid Sensor

Simone Esposito\textsuperscript{a} and Nicholas Devaney\textsuperscript{b}

\textsuperscript{a}Osservatorio Astrofisico di Arcetri, Largo Enrico Fermi 5, Firenze, Italia
\textsuperscript{b}Istituto Astrofisico de Canarias, GTC Project, La Laguna, Tenerife, Spain

ABSTRACT

The high resolution performance of present segmented mirror telescopes like 10m Keck and Gran Telescopio Canarias (GTC) or the future generation of ELT’s like 100m OWL, 50m MAXAT, and 30m CELT will depend critically on the accurate co-phasing of the mirror segments. This paper describes the use of a Pyramid wavefront sensor to detect and correct the segment misalignment. The sensing operation is performed using light from a natural guide star (NGS). We employ simulations which take into account various effects like the atmospheric wavefront disturbances that take place during the sensing process, and wavefront aberrations due to misalignment of the telescope optical train. The results expressed in terms of the segmented mirror rms residual phase are shown and discussed.

1. INTRODUCTION

The image quality of the existing and planned large segmented telescopes, from the two 10m Keck telescopes (J. Nelson, 1997) to the 100m OWL (Dierickx P. & Gilmozzi R., 2000) telescope proposed by ESO community, relies strongly on the precision of the phasing and alignment of the mirror segments. This is especially true for high angular resolution observations. Various techniques have been applied to Keck telescope for segment phasing and alignment. These techniques are modifications of the Shack-Hartman (Chanan et al., 1994) and Curvature sensing (Chanan et al., 1998) approaches to wavefront sensing. They are operated off-line and require the instrument configuration to be changed in order to measure the segment tilts and differential piston. In this paper we show how a Pyramid wavefront sensor (Ragazzoni, 1996, Ragazzoni et al., 1997, Esposito, et al., 2000) can be used to measure differential piston (DP) of the mirror segments. In particular, this technique can measure segment differential piston, segment tip and tilt and low order aberration of the telescope optical train at the same time without changing the sensor configuration. These measurement can be taken using a NGS and averaging the turbulence effects. Moreover if a Pyramid Sensor (PS) is used as the wavefront sensor in an AO system, then the signal from the sensor can be time averaged and used to control the mirror shape during the AO observations. Finally the technique can be upgraded to use three wavelengths so that differential piston larger than \( \pi \) can be measured. Section 2 of this paper summarizes the mirror control scheme starting from the PS signals. Section 3 describes the relationship between a given differential piston and the signal obtained with the PS in further detail. Section 4 reports the results of our active control loop when segments are displaced less than \( \pi \). Section 5 describes how this techniques can be upgraded to measure differential pistons greater than \( \pi \) and finally section 6 report results of the control algorithm when segments are displaced more than \( \pi \). Results detailed in the following show that the active control algorithm using the PS can achieve mirror surface quality of \( \lambda/20 \) or better depending on the operating conditions.

2. PYRAMID SENSOR AND DIFFERENTIAL PISTON INTERACTION MATRIX

The optical configuration of the PS when working in a co-phasing mode is the same configuration as that used in the standard AO mode. However the tip-tilt modulation of the incoming wavefront is no longer required. This simplifies the optical set-up which in this application can be as shown in Fig.1. It has been explained elsewhere (Esposito et al., 2000) that the X or Y PS signal at a point P in the pupil is proportional to a linear combination of the phase differences encountered along an X- or Y-oriented path passing from P. It can therefore be understood that a differential piston in the wavefront will generate a signal. We use diffraction theory and standard FFT algorithm to obtain the pupil images when a certain differential piston is present on the wavefront. In our simulation we considered a segmented telescope having square segments with an entrance pupil shape shown in fig.2. In fig.3 we show, as examples, the sensor signal due to displacing a central segment and an edge segment respectively. The Peak-Valley of the sensor signal as a function of the differential piston amplitude is plotted in fig.4. This shows the sinusoidal relationship between the sensor signal and the
Figure 1. A schematic representation of the basic optical arrangement of the PS.

Figure 2. Left: the pupil geometry of the considered segmented telescope and segment arrangement. Right: A graphical representation of the interaction matrix columns for segment differential piston.

Figure 3. Cross section of the X signal pattern obtained with a central (left) and side (right) actuator.
differential piston. It is this non-linear behavior that prevents the system from measuring differential pistons bigger than \( \pm \pi \). The DP signals obtained by displacing the mirror segments, 52 in our simulation, give the first 52 columns of our interaction matrix while the signals obtained from tip and tilt of each of the segments give another 104 columns. The number of rows in this matrix is, as usual, twice the number of sensing points containing X and Y sensor signals. In our simulation we considered two cases of 64x64 and 16x16 sampling points on the pupil respectively. This correspond to having 8x8 or 2x2 sampling points per segment. This two cases correspond to an active control loop with low frequency cut-off and an adaptive control loop with a standard pupil sampling that can deliver information at a much higher rate respectively. In both cases detector pixels are aligned with the segment edges in the pupil image. The SVD of these two interaction matrices produces very similar eigenvalues confirming that the two sampling gives similar performances in the control loop.

3. DP SIGNAL AND NON LINEARITY

As we found before the relationship between the sensor signal amplitude and the DP amplitude is not linear. This has an impact on the control loop behavior which we discuss below. By using a reconstruction matrix obtained via the SVD from the system interaction matrix we are implicitly assuming a linear relationship between the wavefront disturbances and the sensor signals. In particular, we determine the linear relationship by measuring the derivative of the signal with respect to the amplitude of the input disturbance. This situation is shown in fig. 5 where the real sinusoidal relationship is plotted together with its linear approximation given by a line having a slope equal to the first derivative of the sinus at the origin. This last linear relationship do not allow us to discriminate between DP values \( \phi \) or \( \pi - \phi \). However the correction sign for point 2 is right till this point is below \( \pi \). In a closed loop system this ambiguity will only increase the number of loop iterations by a factor of 2 in order to reach to the zero DP position. So it does not seem to be a big problem for an active control loop. If the point 2 DP is greater then \( \pi \) it is easy to realize that the system will reach a stable position when \( DP = 2\pi \). Considering a generic DP value the linear closed loop correction will drive the DP value to the closest multiple of \( 2\pi \). This feature of the correction loop will be used in the following when dealing with DP bigger than \( \pi \).

4. CO-PHASING WITH \( \|DP\| \leq \pi \)

We report here the results of our simulations when the mirror segment displacements are between \((-\pi/2, \pi/2)\). We consider the cases of having or not having atmospheric turbulence affecting the closed loop operation. The starting point of the simulations is the computation of the system interaction as explained in section 2. The closed loop simulations of DP correction is then carried out. We start applying different DP and tilts plus some low order Zernike aberration to the mirror segments. We then simulate PS operation using two FFTs to generate the pupil images and the sensor signals. Using the reconstruction matrix obtained by inverting the interaction matrix with SVD, we obtain an estimate of the segment displacements and tilts. These measurements are subtracted from the actual segment position and the resulting
Figure 5. Behavior of the closed loop correction for various values of the differential piston.

Figure 6. Left: reduced variance of the wavefront in different atmospheric conditions: triangles $D/r_0 = 0.8$ squares $D/r_0 = 30$. Right: number of iteration of the correction algorithm required to reach the considered wavefront variance.
wavefront is again propagated through the PS. The correction loop is repeated until the wavefront rms is less then \( \lambda/30 \) or \( \lambda/20 \) depending on whether or not turbulence is present. The plot in fig. 6 shows the reduced variance \(^*\) for the considered cases of \( D/r_0 = 0, 8, 30 \). The last two cases correspond to a 10m telescope performing the sensing at about 3\( \mu m \) and 0.8\( \mu m \) respectively. This corresponds to good seeing conditions. Fig. 6 shows the number of iterations that the algorithm requires to converge. When \( D/r_0 \neq 0 \) we integrate 200 realizations per algorithm iteration. Considering a wind speed of 15 m/s and a segment dimension of 1.25m we have a decorrelation time of about 80ms so that each iteration requires 17 sec. From fig. 6 we see that the average number of iteration is about 10 giving about 3 minutes of closed loop operation in order to phase the mirror. In the worst case of about 30 iterations we end up with 10 minutes of closed loop operation.

5. CO-PHASING WITH \( \|DP\| \geq \pi \)

We now consider the case where the DP can be greater than pi. We have to use at least two wavelengths to solve the sign ambiguity and to find the exact amount of DP. In the following we will consider an operating scheme that uses three wavelength. Three wavelengths are used rather than two in order to improve the signal-to-noise ratio (SNR). The two extra wavelengths are obtained by scaling the central wavelength with a single coefficient; if the central wavelength is \( \lambda_1 \), then the other two are obtained as \( \lambda_2 = \lambda_1/\gamma \) and \( \lambda_3 = \lambda_2/\gamma \) (\( \gamma < 1 \)). In the following we describe how to run the closed loop. We run the system using the central wavelength, \( \lambda_1 \). After convergence is achieved the mirror segments will be in a situation shown in fig. 7 where their DP is a multiple of \( \lambda_1 \). In this situation we take a measurement using the two side wavelengths. Using a suitable \( \gamma \) value the two signal obtained will be as shown in fig. 7. It can be seen that the two signal difference are, in this case, positive for positive DP and negative for negative DP. In our closed loop operation we will subtract or add a OPD of one \( \lambda_1 \) accordingly to the sign of the obtained signal difference. By repeating this process will get the system to the zero DP position. It is important to note that this is true only when the considered DP value \( n\lambda_1 \) satisfies the relationship

\[
\frac{n\lambda_1}{\lambda_2} - n \leq 0.5 \tag{1}
\]

or

\[
n(1/\gamma - 1) \leq 0.5 \tag{2}
\]

This sets a limit on the \( \gamma \) value: a value which is too low will limit the dynamic range of the measurement, while a value which is too close to unity will produce a small difference in the two measured signals, so that the SNR of the measurement will be low. The \( \gamma \) value has to be chosen carefully depending on the expected segment DP absolute values and the possible sources of noise (turbulence, noise in the intensity CCD measurements etc.) affecting the measurements. In our simulation the \( \gamma \) value was taken equal to 0.85 this allowing a dynamic range of about 2\( \lambda_1 \). Turbulence effects on the DP signal is not taken into account in this case. The maximum initial value of the DP variance was about 50 rad\(^2\). The results of our simulations are reported in fig. 8. The closed loop operation reaches a \( \lambda/30 \) residual rms in all but two

\(^*\) Throughout the paper, we refer to residual rms or variance of the DP with respect to the mirror.
Figure 8. Left: Wavefront variance obtained after convergence when the initial wavefront contains DP greater then $\pi$. Right: number of the iterations required to achieve the closed loop algorithm convergence in the considered cases.

cases of the 35 wavefront analyzed. The number of iterations required when $DP \geq \pi$ was about 25, as shown in fig. 8. The system failed to converge to zero DP only in two cases. Finally, we note that this algorithm converges even when the DP were all less than $\pi$.

6. CONCLUSION

The use of a Pyramid wavefront sensor to measure the DP of a segmented mirror is demonstrated. Measurements of the DP signals do not require changing the conventional set-up of the pyramid sensor except for the use of a different interaction matrix. The paper shows that the sensor can simultaneously measure the segment tilts, differential piston and continuous wavefront aberrations due to optics misalignment. Our closed loop simulations show that when the differential piston are less that $\pi$ the sensor can drive the segments to an rms less than $\lambda/20$ and $\lambda/30$ in about 10 iteration depending whether turbulence effects are considered or not. The closed loop sensing scheme considered can be upgraded to deal with DP greater than $\pi$. Our simulations show that when turbulence effects are not taken into account, converges to the zero position with an accuracy of $\lambda/30$ in about 25 iteration when the initial DP variance is about 50 rad$^2$. Further simulations has to be performed to study the case of DP rms greater than $\pi$ when turbulence effects are taken into account.

ACKNOWLEDGMENTS

We wish to thanks M. Ricci, A. Riccardi and P. Salinari for useful discussions about the technique presented here.

REFERENCES

P. Dierickx & Gilmozzi R., in SPIE proc. 4004, pp. 290-299, 2000
S. Esposito, O. Feeney, A. Riccardi, in proc. SPIE 4007, pp. 416-422, 2000