High-contrast imaging with ELTs: Effects of cophasing and AO residual errors on the PSF contrast

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ABSTRACT
Direct detection and characterization of terrestrial extrasolar planets are now a high-priority scientific program where new major results from extremely large telescopes (ELTs) are expected. This application is also the most demanding for the adaptive optics (AO) and the mirror segment cophasing. To optimize the fundamental performances of an ELT in high-contrast imaging, we compare the effects of segment cophasing errors with the effects of each AO residual phase errors (wavefront sensor noise, fitting, aliasing, servo-lag) on the long-exposure point-spread function halo. We emphasize that an adaptive correction of the differential segment piston at a nanometric level is needed to keep the contrast gain provided by a high-order AO. We show the potential advantages of an adaptive primary mirror for this purpose. Lastly, we present the planet detection performances in the photon-noise-limited case for different telescopes, AO parameters, and observational conditions (star magnitudes and sites).

Keywords: adaptive optics, extremely large telescopes, exoplanets

1. INTRODUCTION
An adaptive optics (AO) point-spread function (PSF) is characterized by a central core (the diffraction pattern) surrounded by a wide halo containing the residual non-corrected energy (∼1-Strehl), which can hide faint planets. The AO-halo features a bright ring of radius λ/2d, the cutting frequency, where d is the inter-actuator distance (Fig. 1).

A perfect coronagraph in absence of wavefront corrugations (as a Lyot/phase mask with a pupil apodization, FQPM, IAC...) should remove all the coherent light (i.e. the telescope diffraction pattern). Then, only the AO halo remains after coronagraphy and limits the ultimate planet detection performance with its photon noise and speckle noise.

We assume that most of the speckle noise will be removed by the long exposure time needed for exoplanet imaging, and by new promising technics as an image plan wavefront sensing,3 6 or a Synchronous Interferometric Speckle Subtraction.4 We can think that the next generation of ground-based “Planet-Finder” large telescopes will be in the “AO-halo-photon-noise-limited” case.

In this optimal conditions, the planet signal to noise ratio (SNR) scales as the square root of the PSF contrast.7

In this paper, we analyze the AO halo morphology in order to optimize the AO system to the exoplanet imaging. With the same approach, we study the effect from piston errors between the segments of the primary mirror. The comparison of the AO halo and the “pistonic” halo gives the cophasing requirements.

Figure 1 shows the long-exposure seing-limited PSF, the AO PSF and the residual coronagraphic halo (AO halo) in J band of a 30-m segmented ELT equipped with a high-order AO under standard atmospheric conditions and without cophasing errors.

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2. CONTRIBUTIONS OF EACH ADAPTIVE OPTICS ERROR

In order to optimize an AO system, we have decomposed the halo following the four classical errors of an extreme (on-axis) AO system, namely:

1. Wavefront sensor (WFS) noise.
2. Fitting error.
3. Aliasing error.
4. Servo-lag error.

Figure 2 represents the halo profile with the contribution of the four AO error and the cophasing error.

Star magnitude and turbulence have also a big impact on the AO halo. Here, we consider only bright stars with a R-band magnitude between 4 and 8 (sun-like stars at up to 50 pc), and two kind of sites: a good astronomical site as Mauna-Kea ($r_0 \sim 0.2 m$, $\tau_0 \sim 6 ms$), and a theoretically extremely good site as the Dome C in Antarctica ($r_0 \sim 0.3 m$, $\tau_0 \sim 45 ms$).

2.1. WFS noise halo

The WFS noise is the only fundamental limitation (photon + read-out noise) for extreme AO. The noise halo level depends only on the number of photons available per $r_0$ (and not per WFS sub-aperture) during the WFS integration time. The noise halo at $0''1$ is:

$$\text{Noise\ halo at } 0''1 \sim \frac{1}{10} \frac{D^2}{r_0^2} \frac{F_s}{t_i}$$  (1)
Figure 2. Decomposition of the halo following the four classical AO errors (fitting, aliasing, servo-lag, WFS noise) and the cophasing error. Halo profiles are computed from a customization of the PAOLA code.
where $r_0$ is the Fried parameter, $D$ is the diameter of the telescope, $F_s$ is the star flux, and $t_i$ is the integration time. Hence $r_0^2 F_s t_i$ is the number of photons per $r_0$ and per exposure time.

2.2. Fitting error halo

Fitting error is due to a low-pass spatial filtering from the deformable mirror (DM) and the WFS, the associated cutting frequency being $f_c = \lambda/2d$ ($d$ is the actuator or sub-aperture spacing). It is the ultimate limitation, when the AO is noise-free. The normalized mean contrast of the fitting halo is (on-axis):

$$Fitting \text{ halo}|_{\text{on-axis}} \sim 10^{-27} \frac{d^{16/3}}{D^2 \lambda^4 r_0^{10/3}},$$

where $d$ is the actuator or WFS sub-aperture pitch and $\lambda$ the observation wavelength.

2.3. Aliasing error halo

Aliasing error is due to the spatial wavefront sampling by the WFS. A Shack-Hartmann (SH) WFS has a strong aliasing error featuring a square halo (see Fig. 4). The contrast of the SH aliasing halo is on-axis:

$$Aliasing \text{ halo}|_{\text{on-axis}} \sim 1.8 \times 10^{-14} \frac{d^{11/3}}{D^2 \lambda^2 r_0^{5/3}},$$

The aliasing halo is generally dominant but a low-pass spatial filter in the WFS image plan can remove the most part of the SH aliasing error, reaching the naturally low aliasing error of a pyramidal WFS.

2.4. Servo-lag error halo

Servo-lag error is the next limiting error (with the noise), after diffraction pattern and aliasing removing by the coronagraph and the pyramidal WFS respectively. The contrast of the servo-lag halo is at $0''1$ from the axis:

$$Servo - \text{ lag halo}|_{0''1} \sim \frac{w^2 (t_i/2 + t_c)^2}{2 D^2 r_0^{5/3}},$$

where $w$ is the velocity of the dominant turbulent wind, $t_i$ is the WFS integration time and $t_c$ is the CCD readout and wavefront computation delay.

The servo-lag halo shape depends of the wind direction distributions over the turbulent layers, the contrast being better in the perpendicular direction of the dominant layer wind (see Fig. 5).
Figure 4. SH aliasing halo computed numerically (CAOS) and analytically (PAOLA).

Figure 5. Servo-lag halo shape in function of the wind speed and direction.
3. APPLICATION: AO OPTIMIZATION FOR EXOPLANET IMAGING

From this morphological study of the AO-halo, we can easily optimize an extreme AO for planet imaging by adjusting independently the actuator pitch and the WFS integration time:

3.1. Actuator spacing optimization

If aliasing is spatially filtered, the actuator density $d$ is relaxed because there is only the fitting error.

In this favorable case, the actuator spacing is simply adjusted for “digging” a PSF basin just wide enough ($\lambda/2d$) to see the planet (at least 0′.5 for a Jupiter at 10 pc) and deep enough (see fitting error halo expression) to be able to reach the WFS noise limit. A 13–20 cm actuator pitch is optimal for bright stars (i.e. sun-like nearest than 50 pc) from V to K bands (13 cm is for extremely slow turbulence like expected at Dome C, 20 cm is more likely for a site like Mauna Kea).

Figure 6 shows the fitting error halo for various actuator pitches $d$. We can check that the servo-lag halo and the noise halo (both equal in this optimized case) are independent of $d$.

3.2. WFS integration time optimization

The WFS integration time $t_i$ is adjusted in order to equalize the noise halo and the servo-lag halo. If the control delay $t_c$, is a constant fraction of the integration time, the optimal time $t_i^{\text{opt}}$ scales as $r_0^{-1/9} F_s^{-1/3} w_s^{-2/3}$.

This optimization is independent on $D$, $d$, $\lambda$, and is weakly dependent on $r_0$. The optimal integration/delay time is above all dependent on the star magnitude (R-magnitude~ 4–7 for main sequence star from 10 pc to 50 pc) and the wind speed $w$.

Figure 7 shows the halo of an optimized AO for exoplanet searching around nearby stars (sun-like stars at 10 pc) from the Dome C. The same AO system at Mauna-Kea is not yet optimized because of the faster turbulence. A faster AO (more than 1 kHz) would be necessary to reach the WFS noise limit.
Then, it is important to note that high-contrast imaging requires fast AO (1–2 kHz), and/or predictive control,\textsuperscript{8} and/or slow turbulence (like expected at Dome C).

### 4. CONTRIBUTION OF THE COPHASING ERROR

An ELT primary mirror will be necessarily segmented. The diffraction effects due to segmentation (gaps) are static and well determined (PSF calibration and subtraction are possible). But the differential piston and tip-tilt errors are more problematic for high-contrast imaging. From the analytical approach of Yaitskova et al.,\textsuperscript{13} we can write the pistonic-PSF profile\textsuperscript{10}:

$$PSF_{\text{piston}} = \exp\{-\text{rms}^2\} \cdot PSF_{\text{segment}} \cdot GF + \frac{1}{N} (1 - \exp\{-\text{rms}^2\}) \cdot PSF_{\text{segment}},$$

where in particular $\exp\{-\text{rms}^2\}$ gives the pistonic Strehl, $PSF_{\text{segment}} \cdot GF$ is the perfect PSF, and $\frac{1}{N} (1 - \exp\{-\text{rms}^2\}) \cdot PSF_{\text{segment}}$ represents the pistonic halo.

Then, a pistonic halo (equal to the segment PSF) is superimposed on the perfect PSF (or AO-PSF), proportionally to the piston error, degrading the PSF contrast. Figure 8 shows the comparison between different pistonic errors on a 20-m ELT PSF at 1.5 $\mu$m and with hexagonal segments of 0.6 m.

### 5. PISTONIC HALO VS. AO HALO

Figure 9 compares the pistonic halo (for different piston errors value) with the AO halo at Dome C and at Mauna Kea.

The direct comparison of the pistonic halo with the AO halo gives the cophasing requirement. In order to keep the dynamic-range gain provided by the AO, the pistonic halo should be lower or equal to the AO halo. With extreme AO, a piston error lower than $\lambda/300$ rms (5 nm at 1.5 mm) is required. If the piston is greater than $\lambda/100$ rms, we lose also the gain provider by the Dome C.
Figure 8. PSF shape for different pistonic errors and for a 20-m ELT PSF at 1.5 $\mu$m with segments of 0.6 m. Top: cut of the PSFs. Bottom: 2D morphology of the PSFs.
Then, a very fast (i.e. adaptive) and accurate cophasing correction is required to compensate also large stroke perturbations (as wind buffeting) in a nanometric level.

This requires a segmented DM, with the same segmentation than the primary mirror. This will raise difficulties in case of multi-segmentation of various scales (primary+secondary). Hence a segmented adaptive primary mirror\(^{10}\) seems to be well-suited for high-contrast imaging with ELT and extreme AO, because of the high density of actuator and the possibility to correct adaptively the segment piston errors. Let us also note that square segments relax the piston tolerance, as it can be easily deducted from Fig. 10.

6. CONCLUSION

Our decomposition of the halo following each AO error allows fine optimization of extreme AO for high-contrast imaging. We find that the required ingredients for a Planet-Finder telescope are:

1. A high density of actuators and a fast AO \((d \sim 10–20\, \text{cm}, \, f \sim 1–2\, \text{kHz})\).
2. A low-aliasing WFS.
3. An optimized coronagraph to the telescope pupil (diffraction pattern cancelled).
4. A speckle-substraction method.\(^{3,4,6}\)

The comparison of the pistonic halo with the AO halo gives the cophasing requirement in order to keep the AO advantage. A fast adaptive correction of piston seems to be needed to compensate from large stroke perturbations (wind buffeting, vibrations) at nanometric level. A segmented adaptive primary mirror seems to be well-suited.

From optimized AO halo, we can deduce the planet detection performance for any turbulence condition, telescope size and AO parameters.
Figure 11 shows the optimized detection profile (SNR=3 in 10h) in J band of a 15-m ELT located at Mauna-Kea and at Dome C. We can state that:

- From Mauna Kea, a 15-m ELT is enough to detect an exo-Earth (around G to M stars) at 10 pc in 10h in J band.
- From Dome C, the same telescope can detect and analyze all habitable exo-Earths with a planet SNR 4 times higher.
- Finally, from Dome C, the telescope size is like doubled.

For equal performances, a 8-m monolithic telescope at Dome C seems preferable than a 16-m segmented ELT at Mauna-Kea, because of the challenging requirement on the segment cophasing.

Further studies are needed to know if current piston sensors studied and planned for ELTs (pyramid WFS, Mack-Zender interferometer...) can provide an adaptive correction with a nanometric level.

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Figure 11. Optimized detection profile (SNR=3 in 10h) in J band and for a 15-m ELT located at Mauna-Kea and at Dome C. Dotted line is perpendicular to the wind direction (t_i=0.4 ms or 16 ms (perpendicular to the wind) and t_c=0.46 ms).

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